$$\div\,\pi\Big[\tfrac{1}{3}(1-x)^3\sin^{-1}\frac{x}{1-x}-\frac{\pi}{6}(1-x)^3$$

$$+\tfrac{1}{12}(1-2x)^{\frac{1}{2}}+\tfrac{2}{9}(1-2x)^{\frac{3}{2}}-\tfrac{1}{12}(1-2x)^{\frac{5}{2}}\Big]_0^{\frac{1}{2}}$$

$$=\frac{4}{3\pi-4}\left(1-\frac{1}{\pi}-\frac{2}{\pi}\log 2\right).$$

MISCELLANEOUS.

FURTHER REMARK ON PROBLEM 90.

The results of the problem may be put in a better form as follows : e must be a function of s, say $e=f(s)$. For a *continuous* x,

$$f(s+x)=f(s)+x\Delta f(s)+\frac{x(x-1)}{2\,!}\Delta^2 f(s)+\frac{x(x-1)(x-2)}{3\,!}\Delta^3 f(s).$$

Put $s=0$, and substitute for $f(0)$, $\Delta f(0)$, $\Delta^2 f(0)$, $\Delta^3 f(0)$, their values 21, $\frac{7}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, respectively, and

$$f(x)=21+\tfrac{41}{12}x+\frac{x^3}{12};$$

replacing x by a *continuous* s,

$$e=f(s)=21+\frac{41s}{12}+\frac{s^3}{12},$$

which determines once for all the functional relation between any value of s and e. This result is somewhat analagous to the primitive functional relation found from a differential equation, only here difference coefficients enter instead of differential coefficients, and the work is infinitely simpler. Of course the same results could have been obtained by La Grange's formula of interpolation. In order to use the above formula for calculating e for any distance s, 100 yards must be taken as the unit.

E. D. ROE, JR.

93. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Prove that $-(\sqrt{-1})^{\sqrt{-1}}=e^{(\sqrt{-1}-\frac{1}{2})\pi}$.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

In the Napierian system of logarithms we always have $a^n=e^{n\log a}$....(1).